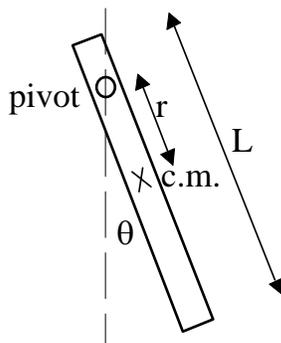# Lab M3: The Physical Pendulum

Another pendulum lab? Not really. This lab introduces <u>angular motion</u>. For the ordinary pendulum, we use Newton's second law, F = ma to describe the motion. For the physical pendulum, we use $\tau = I\,\alpha$, or torque = moment of inertia *times* angular acceleration. We cannot easily use F = ma because the mass is distributed along the length of the pendulum and thus different places on the pendulum have different speeds. Their angular velocity, however, is the same. This lab has the virtue of giving a very good value for g.

A physical pendulum is simply a rigid object which swings freely about some pivot point. The physical pendulum may be compared with a simple pendulum, which consists of a small mass suspended by a (ideally massless) string. In this lab, the physical pendulum is a meter stick with length L = 1m, and it pivots about a fixed point a distance r from the center of mass. The period T of physical pendulum is given by
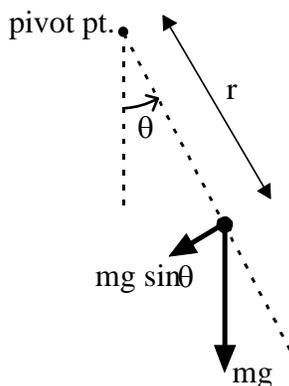

$$(1) \qquad T = 2\pi\sqrt{\frac{I}{mg\,r}}\,,$$

where I is the moment of inertia about the pivot point, m is the total mass, and g is the acceleration of gravity. In this experiment, we will use a physical pendulum to measure g, the acceleration of gravity. With care, it is possible to determine g to within 0.2% or better.

To derive eq'n(1), recall the relation between torque τ and angular acceleration α,

$$(2) \qquad \tau = I\,\alpha\,.$$

In this case, the torque is due to the force of gravity, mg. To compute the torque, one can assume that the full weight mg of the object acts at the center of mass (This is one of the reasons that center-of-mass is such a useful concept.) The magnitude of the torque about the pivot point is $\tau = mgr\sin\theta$. Since $\alpha = \dfrac{d^2\theta}{dt^2}$, we have for the equation of motion (from eq'n (2))

$$(3) \qquad mgr\sin\theta = -I\frac{d^2\theta}{dt^2}\,.$$

Understanding the minus sign in eq'n(3) is a bit tricky.   Torque has a sign, depending on the sense of rotation induced by the torque. If we choose the counterclockwise sense of rotation as positive for both the torque and the angle θ, then at the moment shown in the

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diagram above, the torque is negative and θ is positive and the two always have the opposite sign as the object swings back and forth.

For small θ, $\sin\theta \approx \theta$ (in rads!) , and we can write

(4)
$$\frac{d^2\theta}{dt^2} \cong -\frac{mgr}{I}\theta .$$

This is the equation of a harmonic oscillator; the solution is

(5)
$$\theta(t) = \theta_o \sin(\omega t + \phi) , \qquad \omega = \sqrt{\frac{mgr}{I}} ,$$

and ω is related to the period T by

(6)
$$\omega = \frac{2\pi}{T} , \qquad \text{which leads to eq'n (1).}$$

Although the total mass m appears in the expression (1) for the period T, the moment of inertia I is proportional to m, so the dependence on m cancels out and T is independent of the mass, just as with a simple pendulum. T depends only on the distribution of mass within the object, not on the total mass.

As in Lab M1, measurements of T can be used to compute g, if the moment of inertia I is known. The nice thing about a physical pendulum is that it is mechanically rigid and stable, compared to a simple pendulum (strings can stretch). Notice also that if r in eq'n(1) is small, the period can be made quite large. By contrast, with a simple pendulum, the only way to get a long period is to make the string very long.

The moment of inertia I about the pivot point is related to the moment of inertia about the center of mass I_{cm} by the parallel axis theorem, which states

(7)
$$I = I_{cm} + mr^2.$$

I_{cm} can be computed from the definition of moment of inertia:

(8)
$$I \equiv \sum_i m_i r_i^2 .$$

For a uniform bar of length L, eq'n (8) yields $I_{cm} = \frac{1}{12}m L^2$. Let us call the moment of inertia about the pivot point I_r. We will now manipulate eq'n (1) so that both variables T and r are in the numerator (upstairs). Starting with eq'n(1), we square and use eq'n (7),

$$T^2 \;=\; 4\pi^2\,\frac{I_r}{mg\,r} \;=\; \frac{4\pi^2}{mgr}\Big(I_{cm} + mr^2\Big) \;=\; \frac{4\pi^2}{gr}\left(\frac{I_{cm}}{m} + r^2\right) .$$

For the case of a uniform bar, such as our meter stick, $I_{cm} = \dfrac{1}{12} mL^2$, and we have,

(9) $$\frac{g}{4\pi^2} \cdot T^2 r \;=\; r^2 + \frac{1}{12} L^2.$$

Now let's examine eq'n (9) for a moment. We have gone to some pains to write this in a form which is a linear equation in the variables $T^2 r$ and r^2. Eq'n (9) says that if we measure the period T for several different r's and then we make an x-y plot with the messy quantity $\left(\dfrac{g}{4\pi^2} \cdot T^2 r \right)$ on the y-axis and r^2 on the x-axis, then we should have a straight line with a slope of one and a y-intercept of $\dfrac{1}{12} L^2$. Such a graph, if indeed it has the predicted slope and intercept, would be compelling confirmation of our theoretical calculations.

Equation (9) can be re-arranged slightly to read

(10) $$T^2 r \;=\; \frac{4\pi^2}{g} r^2 + \frac{4\pi^2}{12g} L^2.$$

So if we plot $T^2 r$ vs. r^2, we should get a straight line with slope $= \dfrac{4\pi^2}{g}$. From a measurement of the slope, we can compute g.

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Procedure
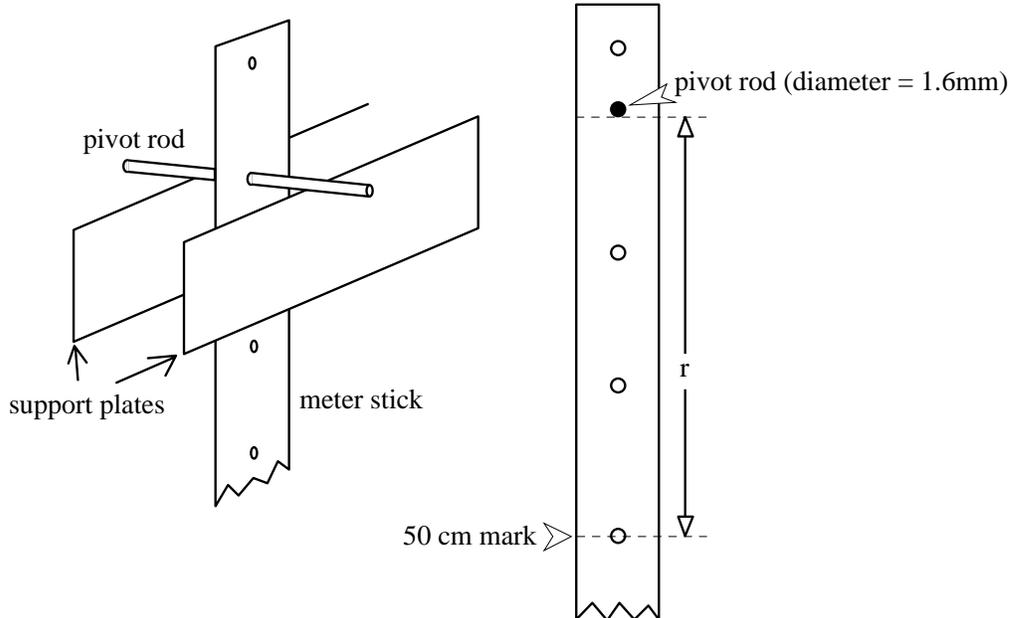
The meter stick has several small holes used for inserting a small pivot rod, which fits snugly in the holes. The distance r is the distance from the center of the meter stick (the 50 cm mark) to the pivot point, the inside edge of the hole; r can be easily determined by noting the distance from the 50 cm mark to the center of the hole and then subtracting 1/2 the diameter of the pivot rod (diameter = 1.6mm).

Using the bubble level provided, check that the pivot rod is level. Also check that the pivot rod is perpendicular to the support plates. Position the photogate near the bottom of the ruler so that the end of the stick blocks the photogate beam when it hangs straight down. Make sure that the photogate is positioned so that it cannot "see" through the pivot holes near the end of the ruler. Keep the amplitude of the swing quite small — just large enough so that the stick swings clear of the beam on either side. After you gently set the stick swinging, let it oscillate for several periods before starting the photogate, in order to allow time for any sideways wobbling motion to damp out.

For each $r > 15 \, \text{cm}$, measure the period T, using the electronic photogate in "period mode".   You should have a total of 8 data points, corresponding to 8 holes. The photogate used with this experiment has the nice feature that it can automatically record several periods and compute the average. Set the photogate to average 10 periods, so that each T you record is really the average of 10 trials. [See the instructions by the photogate. Note: there is no need to stop and restart the pendulum each time you measure the period. Just let it keep swinging while the photogate records several periods.]

Part 1. Check of the theory.

With your data, make a graph of period T vs. the distance r. Then, using the known value of g = 9.796 m/s^2, make a second graph of the quantity $\left(\dfrac{g}{4\pi^2} \cdot T^2 r \right)$ vs. r^2,

and check that the slope of this graph is one and the intercept is $\dfrac{1}{12}L^2 = \dfrac{1}{12}$. You can get the slope and intercept quickly by using the Mathcad functions "slope" and "intercept". (See the instructions near the experiment.) On the same graph, plot the "theory" line $y = mx + b$, where $m = 1$ and $b = 1/12$. Compare your measured values for slope and intercept with the theoretical values.

Part 2. Measurement of g.

Eq'n(10) is

(10)
$$T^2 r \;=\; \frac{4\pi^2}{g} r^2 \;+\; \frac{4\pi^2}{12g} L^2.$$

So now if you plot $T^2 r$ vs. r^2, you should get a line with slope $m = \dfrac{4\pi^2}{g}$ and intercept $b = \dfrac{4\pi^2}{12g} L^2$.   Make this plot. To determine the slope m and intercept b of the best fit line, as well as their uncertainties δm and δb, use the Mathcad document LINFIT.MCD, which is on the hard disk of your computer.   In LINFIT.MCD, all you have to do is enter your x-y data (in this case, $x = r^2$ and $y = T^2 r$) and LINFIT.MCD will do the rest. LINFIT performs a *linear regression* on the data points and finds the line that minimizes the squares of the deviations. Your TA can show you how to open two Mathcad documents at once (your regular lab document and LINFIT) and move between them quickly. You can highlight the entire LINFIT document, then cut and paste it into your lab report. [The file LINFIT.MCD on the hard disk is a read-only file, so you can't accidentally erase or alter the file on the hard disk.]

From your LINFIT values of $m = \dfrac{4\pi^2}{g}$ and $b = \dfrac{4\pi^2}{12g} L^2$, you can make two independent determinations of g: $g_{slope} = 4\pi^2/m$ and $g_{intercept} = 4\pi^2 L^2/12b$ and you can compute their uncertainties from δm and δb. State your fractional uncertainty in g. Compare your measured g with $g_{known} = 9.796 \; m/s^2$. If your g_{meas} and g_{known} do not agree within the uncertainty δg, suggest possible sources of systematic error which might explain the discrepancy.

PreLab Questions:

1. What is the *definition* of torque? (Don't just give a vague, qualitative definition; give the precise, technical definition, as given in physics text books. As usual, if you give an equation, explain what the terms in the equation mean.) Show, using the definition, that the magnitude of the torque on a physical pendulum is $\tau = mgr\sin\theta$. Your answer should include a diagram.

2. Verify that eq'n (5) is a solution of eq'n (4). [Hint: there was a similar problem in the Simple Pendulum Lab M1.]

3. Show that for a uniform rod or bar of length L and mass M, the moment of inertia about an axis through the center of mass and perpendicular to the bar is $I_{cm} = \dfrac{1}{12}M\,L^2$.

[Hint: Use the integral form of eq'n (8), $I = \int \rho\,dr\,r^2$, where $\rho = M/L$ is the mass per unit length (called the linear mass density) of the bar.]

4. Use the result of the question above and the parallel axis theorem [eq'n (7)] to find the moment of inertia of a bar about an axis perpendicular to the bar and through one a point 1/4 of the way from its end.
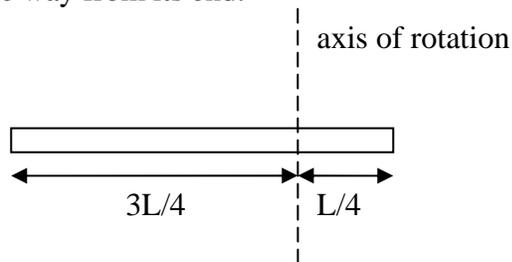
5. In this lab, why is it important to keep the amplitude of motion of the physical pendulum small?

6. In part 2 of this lab, you will make a plot of $T^2 r$ vs. r^2, which should be a straight line with slope $m = \dfrac{4\pi^2}{g}$. From the measured slope m, one can compute g. Derive an expression for δg, the uncertainty in g, in terms of m and δm.

7. Use eq'ns (1) and (7) to compute the period T of physical pendulum consisting of a meter stick (L=1m) with r = 0.30 m. (As always, derive an algebraic expression for T first, then insert numbers to get a numerical value.)

8. For a physical pendulum consisting of a meter stick, plot the period T as a function of the distance r over the range r = 0.10 m to 0.5m. To do this quickly, you should use

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either a programmable calculator or Mathcad. The graph should clearly show the shape of the curve and the location of any maxima or minima.

9. Show with a sketch, what the graph $\left(\dfrac{g}{4\pi^2} \cdot T^2 r\right)$ vs. r^2 should look like. Indicate the values of the slope and intercept.

10. Sketch the graph $T^2 r$ vs. r^2. Indicate the values of the slope and intercept.