

# **NJ ASK Grade 6 Mathematics**

## **Test and Item Specifications**

The purpose of this test is to measure the level of mathematics proficiency that New Jersey students have achieved by spring of the sixth grade. This assessment will reflect the new standards as outlined in the Core Curriculum Content Standards for Mathematics established by New Jersey in July 2002.

### **Standards and Strands**

There are five standards altogether, each of which has a number of lettered strands. These standards, and their associated strands, are enumerated below:

- **4.1. Number and Numerical Operations**
  - A. Number Sense
  - B. Numerical Operations
  - C. Estimation
  
- **4.2. Geometry and Measurement**
  - A. Geometric Properties
  - B. Transforming Shapes
  - C. Coordinate Geometry
  - D. Units of Measurement
  - E. Measuring Geometric Objects
  
- **4.3. Patterns and Algebra**
  - A. Patterns
  - B. Functions and Relationships
  - C. Modeling
  - D. Procedures
  
- **4.4. Data Analysis, Probability, and Discrete Mathematics**
  - A. Data Analysis (Statistics)
  - B. Probability
  - C. Discrete Mathematics—Systematic Listing and Counting
  - D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms
  
- **4.5. Mathematical Processes**
  - A. Problem Solving
  - B. Communication
  - C. Connections
  - D. Reasoning
  - E. Representations
  - F. Technology

The first four of these “standards” also serve as what have been called “*content clusters*” in the previous state assessments; the lettered strands replace what have been called “*macros*” in the directories of test specifications. The fifth standard will continue to provide the “power base” of the assessments.

The student expectations are provided for each strand at Grade 6. The expectations for the fifth standard are intended to address every grade level, so they will remain the same throughout the grades. Since students learn at different rates, narrowing indicators to a single grade level was not always possible; thus indicators at Grades 5 are generally similar to, or modifications of, indicators developed for Grade 6. Teachers at each grade will need to refer to the standards at earlier grade levels to know what topics their students should have learned at earlier grades.

### **A Core Curriculum for Grades K-12**

Implicit in the vision and standards is the notion that there should be a core curriculum for Grades K-12. What does a “core curriculum” mean? It means that every student will be involved in experiences addressing all of the expectations of each of the content standards. It also means that all courses of study should have a common goal of completing this core curriculum, no matter how students are grouped or separated by needs and/or interests.

A core curriculum does not mean that all students will be enrolled in the same courses. Students have different aptitudes, interests, educational and professional plans, learning habits, and learning styles. Different groups of students should address the core curriculum at different levels of depth and should complete the core curriculum according to different timetables. Nevertheless, all students should complete all elements of the core curriculum recommended in the mathematics standards.

All students should be challenged to reach their maximum potential. For many students, the core curriculum described here will indeed be challenging. But if we do not provide this challenge, we will be doing our students a great disservice—leaving them unprepared for the technological and information age of the 21st Century.

For other students, this core curriculum itself will not be a challenge. We have to make sure that we provide these students with appropriate mathematical challenges. We have to make sure that the raised expectations for all students do not result in lowered expectations for our high achieving students. A core curriculum does not exclude a program that challenges students beyond the expectations set in the mathematics standards. Indeed, the vision of equity and excellence calls for schools to provide opportunities for their students to learn more mathematics than is contained in the core curriculum.

***STANDARD 4.1 (NUMBER AND NUMERICAL OPERATIONS)***  
**All students will develop number sense and will perform standard numerical operations and estimations on all types of numbers in a variety of ways.**

## **Descriptive Statement**

Numbers and arithmetic operations are what most of the general public think about when they think of mathematics; and even though other areas, such as geometry, algebra, and data analysis, have become increasingly important in recent years, numbers and operations remain at the heart of mathematical teaching and learning. Facility with numbers, the ability to choose the appropriate types of numbers and the appropriate operations for a given situation, and the ability to perform those operations as well as to estimate their results are all skills essential for modern day life.

**Unless otherwise noted, all indicators for Grade 6 pertain to these sets of numbers:**

- All Integers
- All fractions as part of a whole, as a subset of a set, as a location on a number line, and as division of whole numbers.
- All decimals.

## **Strands**

### **4.1.A. Number Sense**

Number sense is an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent that comes from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how each can best be used to describe a particular situation. It subsumes the more traditional category of school mathematics curriculum called numeration and thus includes the important concepts of place value, number base, magnitude, approximation, and estimation.

### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

### **Stimulus Characteristics**

- For whole numbers, test items may include illustrations of the following: thousands blocks, hundreds blocks, tens blocks, ones blocks; unifix cubes, sticks, number lines; other counting manipulatives.
- For fractions or fractional parts, items may include illustrations of real-life objects or simple geometric shapes, such as circles or rectangles.
- Number lines may be used when asking students to order and compare the magnitude of whole numbers, decimals and/or fractions.
- The symbols =, <, and > may be used to compare whole numbers to whole numbers, fractions to fractions, whole numbers to fractions, whole numbers to decimals, and decimals to decimals.

### **Item Guidelines**

- 4.1.A.1** Use real-life experiences, physical materials, and technology to construct meanings for numbers
- 4.1.A.2** Recognize the decimal nature of United States currency and compute with money.
- 4.1.A.3** Demonstrate a sense of the relative magnitudes of numbers
- 4.1.A.4** Explore the use of ratios and proportions in a variety of situations.
- 4.1.A.5** Understand and use whole-number percents between 1 and 100 in a variety of situations.
- 4.1.A.6** Use whole numbers, fractions, and decimals to represent equivalent forms of the same number.
- 4.1.A.7** Develop and apply number theory concepts in problem solving situations.
- 4.1.A.8** Compare and Order Numbers.

### **4.1.B. Numerical Operations**

Numerical operations are an essential part of the mathematics curriculum, especially in the elementary grades. Students must be able to select and apply various computational methods, including mental math, pencil-and-paper techniques, and the use of calculators. Students must understand how to add, subtract, multiply, and divide whole numbers,

fractions, decimals, and other kinds of numbers. With the availability of calculators that perform these operations quickly and accurately, the instructional emphasis now is on understanding the meanings and uses of these operations and on estimation and mental skills, rather than solely on the development of paper-and-pencil proficiency.

### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

### **Stimulus Characteristics**

- Extended-response items may show pictures and have students mathematically model what they see in the illustrations.
- Computation items may appear either in a vertical or horizontal format.

### **Item Guidelines**

- 4.1.B.1** Recognize the appropriate use of each arithmetic operation in problem situations.
- 4.1.B.2** Construct, use, and explain procedures for performing calculations with fractions and decimals.
- 4.1.B.3** Use an efficient and accurate pencil-and-paper procedure for division of a 3-digit number by a 2-digit number.
- 4.1.B.4** Select pencil-and-paper, mental math, or a calculator as the appropriate computational method in a given situation depending on the context and numbers.
- 4.1.B.5** Find squares and cubes of whole numbers
- 4.1.B.6** Check the reasonableness of results of computations.
- 4.1.B.7** Understand and use the various relationships among operations and properties of operations.
- 4.1.B.8** Understand and apply the standard algebraic order of operations for the four basic operations, including appropriate use of parentheses.

### **4.1.C. Estimation**

Estimation is a process that is used constantly by mathematically capable adults and one that can be mastered easily by children. It involves an educated guess about a quantity or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that determination is through the use of strong estimation skills.

Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact one. Students can learn to make these judgments and use mathematics more powerfully as a result.

#### **Item Format**

Multiple-choice and constructed response items may be used to test appropriateness of estimation. Extended-response items may be used to test estimation used for predicting or for determining reasonableness of answers.

#### **Stimulus Characteristics**

Charts, tables, diagrams, and illustrations may be used.

#### **Item Guidelines**

- 4.1.C.1** Use a variety of strategies for estimating both quantities and the results of computations.
- 4.1.C.2** Recognize when an estimate is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.
- 4.1.C.3** Determine the reasonableness of an answer by estimating the result of operations.
- 4.1.C.4** Determine whether a given estimate is an overestimate or an underestimate.

***STANDARD 4.2 (GEOMETRY AND MEASUREMENT)***

**All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe, and analyze phenomena.**

## **Descriptive Statement**

Spatial sense is an intuitive feel for shape and space. Geometry and measurement both involve describing the shapes we see all around us in art, nature, and the things we make. Spatial sense, geometric modeling, and measurement can help us to describe and interpret our physical environment and to solve problems.

## **Strands**

### **4.2.A. Geometric Properties**

This includes identifying, describing, and classifying standard geometric objects, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.

### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

### **Stimulus Characteristics**

- Stimulus may include illustrations of two-dimensional figures, three-dimensional figures, and real-life objects, when appropriate.
- Pictorial representations of symmetrical, congruent, or similar figures may be used in items.

### **Item Guidelines**

**4.2.A.1** Understand and apply concepts involving lines and angles.

**4.2.A.2** Identify, describe, compare, and classify polygons and circles.

- 4.2.A.3 Identify similar figures.
- 4.2.A.4 Understand and apply the concepts of congruence and symmetry (line and rotational).
- 4.2.A.5 Compare properties of cylinders, prisms, cones, pyramids, and spheres.
- 4.2.A.6 Identify, describe, and draw the faces or shadows (projections) of three-dimensional geometric objects from different perspectives.
- 4.2.A.7 Identify a three-dimensional shape with given projections (top, front and side views).
- 4.2.A.8 Identify a three-dimensional shape with a given net (i.e., a flat pattern that folds into a 3D shape).

#### **4.2.B. Transforming Shapes**

Analyzing how various transformations affect geometric objects allows students to enhance their spatial sense. This includes combining shapes to form new ones and decomposing complex shapes into simpler ones. It includes the standard geometric transformations of translation (slide), reflection (flip), rotation (turn), and dilation (scaling). It also includes using tessellations and fractals to create geometric patterns.

#### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

#### **Stimulus Characteristics**

- Geometric transformations that appear in items may or may not be illustrated on a grid.
- Grids will be provided in items that require students to draw a particular geometric transformation.
- Location grids may use ordered positive number pairs or letter and number combinations.

#### **Item Guidelines**

- 4.2.B.1** Use a translation, a reflection, or a rotation to map one figure onto another congruent figure.
  
- 4.2.B.2** Recognize, identify, and describe geometric relationships and properties, as they exist in nature, art, and other real-world settings.

### **4.2.C. Coordinate Geometry**

Coordinate geometry provides an important connection between geometry and algebra. It facilitates the visualization of algebraic relationships, as well as an analytical understanding of geometry.

#### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

#### **Stimulus Characteristics**

- Real world context may include maps, grids, or number lines.
- *Only* Quadrant I from Cartesian planes may be used.
- Grids will be provided in items that require students to plot a particular geometric figure.
- Location grids may use ordered number pairs or letter and number combinations.
- All grids will have the origin and scales labeled.

#### **Item Guidelines**

**4.2.C.1** Create geometric shapes with specified properties in the first quadrant on a coordinate grid.

### **4.2.D. Units of Measurement**

Measurement helps describe our world using numbers. An understanding of how we attach numbers to real-world phenomena, familiarity with common measurement units (e.g., inches, liters, and miles per hour), and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.

#### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

#### **Stimulus Characteristics**

- Pictures, diagrams, charts, tables and graphs may be used in items, when appropriate.
- Examples of possible pictorial stimuli include meter sticks, rulers, analog and digital timepieces, thermometers, balances, scales, coins, bills, two-dimensional representations of objects with volume/capacity (rectangular only).

### **Item Guidelines**

- 4.2.D.1** Select and use appropriate units to measure angles, area, surface area, and volume.
- 4.2.D.2** Use a scale to find a distance on a map or a length on a scale drawing.
- 4.2.D.3** Convert measurement units within a system (e.g., 3 feet = \_\_\_ inches).
- 4.2.D.4** Know approximate equivalents between the standard and metric systems (e.g., one kilometer is approximately 6/10 of a mile).
- 4.2.D.5** Use measurements and estimates to describe and compare phenomena.

### **4.2.E. Measuring Geometric Objects**

This area focuses on applying the knowledge and understanding of units of measurement in order to actually perform measurement. While students will eventually apply formulas, it is important that they develop and apply strategies that derive from their understanding of the attributes. In addition to measuring objects directly, students apply indirect measurement skills, using, for example, similar triangles and trigonometry.

### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

### **Stimulus Characteristics**

- Pictorial representations of measuring devices may appear adjacent to the object that is to be measured.
- Pictorial representations of measuring devices will reflect actual measures.

### **Item Guidelines**

- 4.2.E.1** Use a protractor to measure angles.

- 4.2.E.2** Develop and apply strategies and formulas for finding perimeter and area.
- 4.2.E.3** Develop and apply strategies and formulas for finding the surface area and volume of rectangular prisms and cylinders.
- 4.2.E.4** Recognize that shapes with the same perimeter do not necessarily have the same area and vice versa.
- 4.2.E.5** Develop informal ways of approximating the measures of familiar objects (e.g., use a grid to approximate the area of the bottom of one's foot).

**STANDARD 4.3 (PATTERNS AND ALGEBRA)**

**All students will represent and analyze relationships among variable quantities and solve problems involving patterns, functions, and algebraic concepts and processes.**

**Descriptive Statement**

Algebra is a symbolic language used to express mathematical relationships. Students need to understand how quantities are related to one another, and how algebra can be used to concisely express and analyze those relationships. Modern technology provides tools for supplementing the traditional focus on algebraic procedures, such as solving equations, with a more visual perspective, with graphs of equations displayed on a screen. Students can then focus on understanding the relationship between the equation and the graph, and on what the graph represents in a real-life situation.

**Strands**

**4.3.A. Patterns**

Algebra provides the language through which we communicate the patterns in mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and, by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena.

**Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

**Item Guidelines**

**4.3.A.1** Recognize, describe, extend, and create patterns involving whole numbers and rational numbers.

### **4.3.B. Functions and Relationships**

The function concept is one of the most fundamental unifying ideas of modern mathematics. Students begin their study of functions in the primary grades, as they observe and study patterns. As students grow and their ability to abstract matures, students form rules, display information in a table or chart, and write equations that express the relationships they have observed. In high school, they use the more formal language of algebra to describe these relationships.

#### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

#### **Item Guidelines**

**4.3.B.1** Describe the general behavior of functions given by formulas or verbal rules (e.g., graph to determine whether increasing or decreasing, linear or not).

### **4.3.C. Modeling**

Algebra is used to model real-life situations and answer questions about them. This use of algebra requires the ability to represent data in tables, pictures, graphs, equations or inequalities, and rules. Modeling ranges from writing simple number sentences to help solve story problems in the primary grades to using functions to describe the relationship between two variables, such as the height of a pitched ball over time. Modeling also includes some of the conceptual building blocks of calculus, such as how quantities change over time and what happens in the long run (limits).

#### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

#### **Stimulus Characteristics**

- Charts or tables showing number patterns may be used.
- Word problems may be used.
- Items may include the use of variables or symbols (e.g., an empty box) to represent unknown quantities in equations.

### **Item Guidelines**

**4.3.C.1** Use patterns, relations, and linear functions to model situations.

**4.3.C.2** Draw freehand sketches of graphs that model real phenomena and use such graphs to predict and interpret events.

### **4.3.D. Procedures**

Techniques for manipulating algebraic expressions—procedures—remain important, especially for students who may continue their study of mathematics in a calculus program. Utilization of algebraic procedures includes understanding and applying properties of numbers and operations, using symbols and variables appropriately, working with expressions, equations, and inequalities, and solving equations and inequalities.

### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

### **Stimulus Characteristics**

- Charts or tables showing number patterns may be used.
- Word problems may be used.

### **Item Guidelines**

**4.3.D.1** Solve simple linear equations with manipulatives and informally

**4.3.D.2** Understand and apply the properties of operations and numbers.

**4.3.D.3** Evaluate numerical expressions

**4.3.D.4** Extend understanding and use of inequality.

**STANDARD 4.4 (DATA ANALYSIS, PROBABILITY,  
AND DISCRETE MATHEMATICS)**

**All students will develop an understanding of the concepts and techniques of data analysis, probability, and discrete mathematics and will use them to model situations, solve problems, and analyze and draw appropriate inferences from data.**

## **Descriptive Statement**

Data analysis, probability, and discrete mathematics are important interrelated areas of applied mathematics. Each provides students with powerful mathematical perspectives on everyday phenomena and with important examples of how mathematics is used in the modern world. Two important areas of discrete mathematics are addressed in this standard; a third area, iteration and recursion, is addressed in Standard 4.3 (Patterns and Algebra).

These topics provide students with insight into how mathematics is used by decision-makers in our society and with important tools for modeling a variety of real-world situations. Students will better understand and interpret the vast amounts of quantitative data that they are exposed to daily, and they will be able to judge the validity of data-supported arguments.

## **Strands**

### **4.4.A. Data Analysis (Statistics)**

In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.

### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

### **Stimulus Characteristics**

- Pictorial representations (e.g., icons), graphs, tables, and charts may be used.
- Graphs may include pictographs, bar graphs, line plots, and simple line graphs.
- Sets of data in a meaningful context may be used in test items.

### **Item Guidelines**

**4.4.A.1** Collect, generate, organize, and display data.

**4.4.A.2** Read, interpret, select, construct, analyze, generate questions about, and draw inferences from displays of data.

**4.4.A.3** Respond to questions about data, generate their own questions and hypotheses, and formulate strategies for answering their questions and testing.

### **4.4.B. Probability**

Students need to understand the fundamental concepts of probability so that they can interpret weather forecasts, avoid unfair games of chance, and make informed decisions about medical treatments whose success rate is provided in terms of percentages. They should regularly be engaged in predicting and determining probabilities, often based on experiments (such as flipping a coin 100 times), but eventually based on theoretical discussions of probability that make use of systematic counting strategies. High school students should use probability models and solve problems involving compound events and sampling.

### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

### **Item Guidelines**

**4.4.B.1** Determine probabilities of events.

**4.4.B.2** Determine probability using intuitive, experimental, and theoretical methods (e.g., using model of picking items of different colors from a bag).

**4.4.B.3** Explore compound events.

**4.4.B.4** Model situations involving probability using simulations (with spinners, dice) and theoretical models.

**4.4.B.5** Recognize and understand the connections among the concepts of independent outcomes, picking at random, and fairness

#### **4.4.C. Discrete Mathematics—Systematic Listing and Counting**

Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor

#### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

#### **Item Guidelines**

**4.4.C.1** Solve counting problems and justify that all possibilities have been enumerated without duplication.

**4.4.C.2** Apply the multiplication principle of counting.

**4.4.C.3** List the possible combinations of two elements chosen from a given set (e.g., forming a committee of two from a group of 12 students, finding how many handshakes there will be among ten people if everyone shakes each other person's hand once).

#### **4.4.D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms**

Vertex-edge graphs, consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve problems based on real-world situations. Students should learn to follow and devise lists of instructions, called “*algorithms*,” and use algorithmic thinking to find the best solution to problems like those involving vertex-edge graphs, but also to solve other problems.

### **Item Format**

Multiple-choice and constructed response items may be used to test this strand unless otherwise noted; answer choices may be symbolic or pictorial.

### **Item Guidelines**

- 4.4.D.1** Devise strategies for winning simple games (e.g., start with two piles of objects, each of two players in turn removes any number of objects from a single pile, and the person to take the last group of objects wins) and express those strategies as sets of directions.
- 4.4.D.2** Analyze vertex-edge graphs and tree diagrams.
- 4.4.D.3** Use vertex-edge graphs to find solutions to practical problems.

***STANDARD 4.5 (MATHEMATICAL PROCESSES)***

**All students will use mathematical processes of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.**

## **Descriptive Statement**

The mathematical processes described here highlight ways of acquiring and using the content knowledge and skills delineated in the first four mathematical standards.

## **Strands**

### **4.5.A. Problem Solving**

Problem posing and problem solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. Through problem solving, students experience the power and usefulness of mathematics. Problem solving is interwoven throughout the grades to provide a context for learning and applying mathematical ideas.

### **Item Guidelines**

- 4.5.A.1** Learn mathematics through problem solving, inquiry, and discovery.
- 4.5.A.2** Solve problems that arise in mathematics and in other contexts.
- Open-ended problems
  - Non-routine problems
  - Problems with multiple solutions
  - Problems that can be solved in several ways
- 4.5.A.3** Select and apply a variety of appropriate problem-solving strategies (e.g., try a simpler problem, make a diagram) to solve problems.
- 4.5.A.4** Pose problems of various types and levels of difficulty.

**4.5.A.5** Monitor their progress and reflect on the process of their problem-solving activity.

### **4.5.B. Communication**

Communication of mathematical ideas involves students' sharing their mathematical understandings in oral and written form with their classmates, teachers, and parents. Such communication helps students clarify and solidify their understanding of mathematics and develop confidence in themselves as mathematics learners. It also enables teachers to better monitor student progress.

#### **Item Guidelines**

**4.5.B.1** Use communication to organize and clarify their mathematical thinking.

- Reading and writing
- Discussion, listening, and questioning

**4.5.B.2** Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.

**4.5.B.3** Analyze and evaluate the mathematical thinking and strategies of others.

**4.5.B.4** Use the language of mathematics to express mathematical ideas precisely.

### **4.5.C. Connections**

Making connections involves seeing relationships between different topics and drawing on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to daily life.

#### **Item Guidelines**

**4.5.C.1** Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).

- 4.5.C.2** Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
- 4.5.C.3** Recognize that mathematics is used in a variety of contexts outside mathematics.
- 4.5.C.4** Apply mathematics in practical situations and in other disciplines.
- 4.5.C.5** Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
- 4.5.C.6** Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

#### **4.5.D. Reasoning**

Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied.

#### **Item Guidelines**

- 4.5.D.1** Recognize that mathematical facts, procedures, and claims must be justified.
- 4.5.D.2** Use reasoning to support their mathematical conclusions and problem solutions.
- 4.5.D.3** Select and use various types of reasoning and methods of proof.
- 4.5.D.4** Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
- 4.5.D.5** Make and investigate mathematical conjectures.
- Counterexamples as a means of disproving conjectures

- Verifying conjectures using informal reasoning or proofs

**4.5.D.6** Evaluate examples of mathematical reasoning and determine whether they are valid.

#### **4.5.E. Representations**

Representations refers to the use of physical objects, drawings, charts, graphs, and symbols to represent mathematical concepts and problem situations. By using various representations, students will be better able to communicate their thinking and solve problems. Using multiple representations will enrich the problem solver with alternative perspectives on the problem. Historically, people have developed and successfully used manipulatives (concrete representations such as fingers, base-ten blocks, geoboards, and algebra tiles) and other representations (such as coordinate systems) to help them understand and develop mathematics.

#### **Item Guidelines**

**4.5.E.1** Create and use representations to organize, record, and communicate mathematical ideas.

- Concrete representations (e.g., base-ten blocks or algebra tiles)
- Pictorial representations (e.g., diagrams, charts, or tables)
- Symbolic representations (e.g., a formula)
- Graphical representations (e.g., a line graph)

**4.5.E.2** Select, apply, and translate among mathematical representations to solve problems.

**4.5.E.3** Use representations to model and interpret physical, social, and mathematical phenomena.

#### **4.5.F. Technology**

Calculators and computers need to be used along with other mathematical tools by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Students should explore both new and familiar concepts with calculators and computers and also should become proficient in using technology as it is used by adults (e.g., for assistance in solving real-world problems).

#### **Item Guidelines**

- 4.5.F.1** Use technology to gather, analyze, and communicate mathematical information.
  
- 4.5.F.2** Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information (cf. workplace readiness Standard 8.4-D).
  
- 4.5.F.3** Use graphing calculators and computer software to investigate properties of functions and their graphs.
  
- 4.5.F.4** Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
  
- 4.5.F.5** Use computer software to make and verify conjectures about geometric objects.
  
- 4.5.F.6** Use computer-based laboratory technology for mathematical applications in the sciences (cf. science standards).

## **General Item Specifications**

There are some general considerations and procedures that make the task of item development more efficient and effective. These considerations include, but are not limited to, the following:

- Items attempt to focus on content that is “real world” and is familiar to sixth grade students.
- Item stems and stimulus materials are straightforward and use simple syntax.
- Items must include language that is unbiased and that will not disadvantage a particular group of students.
- Items do not display demeaning representations of gender or ethnic groups, persons with disabilities, cultural or religious groups, nor do items contain elements that might offend any such groups.
- Items will not test vocabulary items explicitly.
- Complex directions in items will be organized with bullets to identify multiple requirements.

## **Rules for Multiple-Choice Items**

All item stems must clearly indicate what is expected in a response and must help students focus their response.

- Each multiple-choice item will have a stem (question, statement, or incomplete statement) and four answer (or completion) options, only one of which is correct. Correct answers will be distributed as evenly as possible among As, Bs, Cs and Ds.
- Multiple-choice item stems will present a complete problem so that students will know what to do before looking at the answer choices. Students should not need to read all answer choices before knowing what is expected.
- The four answer choices will be approximately the same length, will have the same format, and will be syntactically and semantically parallel; students should not be able to rule out a wrong answer or identify a correct response simply by virtue of superficial or trivial characteristics, syntactic complexity, or concept complexity.
- Correct answers should show what students know and are able to do; correct answers should not be obtainable through incorrect procedures or gimmicks.
- Distractors (wrong answer choices) will reflect common errors or misunderstandings, naïve pre-conceptions, or other misconceptions so that correct responses show what students know or can do rather than students simply eliminating incorrect responses by virtue of a distractor's obviously inappropriate nature.
- Distractors will not be partially correct responses, nor will they be designed to “trick” students into responding incorrectly.

## **Rules for Extended-Response Items**

Each extended-response item will give clear indications of what is required of students (e.g., if two words are required, the stem will indicate this; if a number sentence is required, the stem will indicate this).

Anything required by the scoring rule will be asked for in the item stem.

- Item stems will be succinct with simple syntax and familiar words.
- Item responses will be carefully scaffolded to guide students' responses; for example, a response that requires multiple parts will be presented with bullets to draw attention to the parts.
- Extended-response items will require responses that range in length from several numbers to three to four steps.
- Extended-response items may ask for a graph/figure/diagram/table with labels or a figure/diagram/table with one or two words, sentences, or number sentences to support the figure/diagram/table.
- Any extended-response item that requires students to use information from a stimulus will specifically ask for the information (e.g., which numbers in the table could you use to..., support your response with specific information from...).
- Extended-response items will require a more complex process to develop a viable solution, demonstrate an understanding or process, communicate mathematical ideas or results, or show reasoning.
- Extended-response items may ask students to organize information, explain choices, support claims, or verify arguments, conclusions, solutions, and thinking.

## **Rules for Stimulus Materials**

- Stimulus materials may include graphs, charts, tables, diagrams, figures, brief passages, number sentences, and other mathematical communications relevant to grade 6.
- Stimulus materials should be clean and simple with a minimum of distracting or irrelevant information. Irrelevant information should be included only when the target being assessed is focused on extraction of relevant information and/or identification of irrelevant information.
- Stimulus materials should not “trick” students into choosing or developing an incorrect or ineffective response.
- Reading level for stimulus materials should be at least one grade below the sixth-grade level.

## **Rules for Developing Scoring Rubrics**

- An item-specific scoring rubric will be used for each extended-response item.
- Information from student sample papers will be used to refine these scoring rubrics.
- Scoring rubrics will follow a “focused holistic” model in which the score for the response is based on overall quality in relation to the strand; individual score point elements must arise from the specific individual requirements of the item and its target.
- Scoring rubrics for content strand items will focus on conceptual understanding, application of appropriate procedures, and accuracy.
- Scoring rubrics for process items will focus on such aspects as clarity, effectiveness, reasonableness, selection of useful procedures, and/or degree to which solutions are viable.
- Extended-response items will be scored with a four-level scoring rubric (0-3).

The following matrices demonstrate how the operational form for NJ ASK Grade 6 Mathematics was constructed.

Test Construction Map for NJ ASK Grade 6 Mathematics

<b>Standard</b>	<b>Specified MC</b>	<b>Actual (1 pt.)</b>	<b>Specified OE</b>	<b>Actual (3 pts.)</b>	<b>Total Items</b>	<b>Total Points</b>
I	6–10		0–1			
II	6–10		0–1			
III	6–10		0–1			
IV	6–10		0–1			
<b>Total Items</b>	<b>30</b>		<b>3</b>		<b>33</b>	
<b>Total Points</b>	<b>30</b>		<b>9</b>			<b>39</b>

Actual Test Map for 2006 NJ ASK Grade 6 Mathematics

<b>Standard</b>	<b>Specified MC</b>	<b>Actual (1 pt.)</b>	<b>Specified OE</b>	<b>Actual (3 pts.)</b>	<b>Total Items</b>	<b>Total Points</b>
I	6–10	9	0–1	0	9	9
II	6–10	7	0–1	1	8	10
III	6–10	7	0–1	1	8	10
IV	6–10	7	0–1	1	8	10
<b>Total Items</b>	<b>30</b>	<b>30</b>	<b>3</b>	<b>3</b>	<b>33</b>	
<b>Total Points</b>	<b>30</b>	<b>30</b>	<b>9</b>	<b>9</b>		<b>39</b>

The set of sample items that follow demonstrate the style and rigor students can expect on the NJ ASK Grade 6 Mathematics assessment. One multiple-choice item and one open-ended item are represented. Each item is aligned to the indicator level of the Core Curriculum Content Standards and to the Mathematical Processes standard, or “power base,” as follows:

- A indicates the item aligns to 4.5.A, or the Problem Solving strand.
- B indicates the item aligns to 4.5.B, or the Communication strand.
- C indicates the item aligns to 4.5.C, or the Connections strand.
- D indicates the item aligns to 4.5.D, or the Reasoning strand.
- E indicates the item aligns to 4.5.E, or the Representations strand.
- F indicates the item aligns to 4.5.F, or the Technology strand.

Each item can be aligned to (1) no strands, or (2) up to and including all of the strands within the mathematical processes standard. The power base is indicated by letters for the strands that apply and asterisks for the strands that do not apply. Therefore a power base of an item that only aligns to 4.5.D, or Reasoning, would be written, “\*\*\*\*E\*”.

## Sample Item 1

Tamara tells a friend that she can multiply 2-digit numbers in her head using the distributive property. She gives the following example to her friend.

$$\begin{aligned}18 \times 12 &= (18 \times 10) + (18 \times 2) \\ &= 180 + 36 \\ &= 216\end{aligned}$$

Using Tamara's method, how could you multiply  $32 \times 15$ ?

- A.  $(30 \times 10) + (20 \times 5)$
- B.  $(32 \times 10) + (32 \times 5)$
- C.  $(30 \times 10) + (2 \times 5)$
- D.  $(30 \times 5) + (2 \times 10)$

The correct answer is B.

This item aligns to 4.3.D.2: Understand and apply the properties of operations and numbers.

The power base for this item is \*B\*\*E\*.

## Sample Item 2

Rashad has an aquarium full of fish. In his aquarium, 25% of the fish are orange, 50% are striped, and 25% are black. If Rashad has 4 orange fish, how many of his fish are striped? What is the total number of fish in the aquarium?

Show your work here.

Striped fish: \_\_\_\_\_ Total fish: \_\_\_\_\_

This item is scored using a 3-point rubric.

This item aligns to 4.1.A.5: Understand and use whole-number percents between 1 and 100 in a variety of situations.

The power base for this item is A\*\*\*\*\*.