GRADE EIGHT INDICATORS

FOR THE

MATHEMATICS

STANDARD COURSE OF
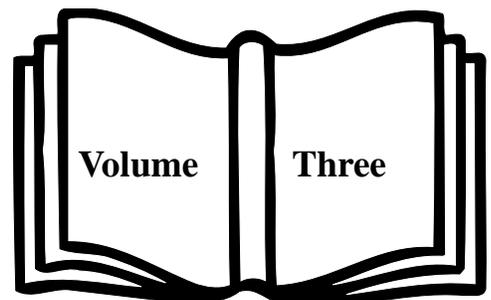
STUDY

Volume   **Three**

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The Indicators for Grade Eight Mathematics

What are Indicators?

Indicators are measures to determine mastery of a concept, procedure, or application within a specific objective or group of objectives. The Indicators illustrate and elaborate each objective with sample problems and tasks, vocabulary, and related concepts and skills. They are written to provide a fuller explanation of the objectives in the Grade Eight Mathematics Standard Course of Study. Whenever possible they are couched in a context to further illustrate the scope of the objectives. Indicators are summative in nature, that is, they are intended to show the kind of mathematical problem solving that is appropriate to indicate a student's mastery of the curriculum after an extended period of instruction and practice.

The items contained in this document are not intended to represent sample end-of-grade test questions. Students are encouraged to explain or defend their responses and not merely give an answer. Communication is an important part of mathematics and mathematics education. Writing in mathematics helps students solidify their thinking and gives teachers an insight into the thought process of their students.

It is hoped that teachers will find this material useful in understanding both the intent of the 2003 revised Mathematics Standard Course of Study and the thinking of their students.

Questions and comments should be directed to Linda Patch at the Department of Public Instruction (lpatch@dpi.state.nc.us or 919.807.3841).

Grade 8

Number and Operations

1.01 Develop number sense for the real numbers.
a) Define and use irrational numbers.
b) Compare and order.
c) Use estimates of irrational numbers in appropriate situations.

1.02 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

Measurement

2.01 Determine the effect on perimeter, area or volume when one or more dimensions of two- and three-dimensional figures are changed.

2.02 Apply and use concepts of indirect measurement.

Geometry

3.01 Represent problem situations with geometric models.

3.02 Apply geometric properties and relationships, including the Pythagorean theorem, to solve problems.

3.03 Identify, predict, and describe dilations in the coordinate plane.

Data Analysis and Probability

4.01 Collect, organize, analyze, and display data (including scatterplots) to solve problems.

4.02 Approximate a line of best fit for a given scatterplot; explain the meaning of the line as it relates to the problem and make predictions.

4.03 Identify misuses of statistical and numerical data.

Algebra

5.01 Develop an understanding of function.
a) Translate among verbal, tabular, graphic, and algebraic representations of functions.
b) Identify relations and functions as linear or nonlinear.
c) Find, identify, and interpret the slope (rate of change) and intercepts of a linear relation.
d) Interpret and compare properties of linear functions from tables, graphs, or equations.

5.02 Write an equation of a linear relationship given: two points, the slope and one point on the line, or the slope and y-intercept.

5.03 Solve problems using linear equations and inequalities, justify symbolically and graphically.

5.04 Solve equations using the inverse relationships of addition and subtraction, multiplication and division, squares and square roots, and cubes and cube roots.

1.01   Develop number sense for the real numbers.

a) Define and use irrational numbers.
b) Compare and order.
c) Use estimates of irrational numbers in appropriate situations.

To achieve this objective, students should be able to:

- *Understand and know that irrational numbers are numbers that cannot be expressed as the ratio or quotient of two integers.*

- *Determine whether radical expressions represent rational or irrational numbers.*

- *Simplify radical expressions.*

- *Approximate the value of irrational numbers between two consecutive integers.*

A.   A square kite has an area of 5 square feet. To the nearest tenth of a foot, what is the length of one side of the kite?

B.   During the investigation of traffic accidents, a piece of evidence that is identified and analyzed is the skid marks left by the cars involved. The formula $S = \sqrt{30df}$ is used to determine the vehicle's minimim speed, S in mph, where d = length of the skid marks in feet and f = drag factor. A recent accident at an intersection showed skid marks for the two cars involved to be lengths of 36′ 7″ and 42′ 7″. Find the minimum speed of each car if the drag factor for both vehicles was 0.83.

radical

terminating decimal
repeating decimal
non-terminating
non-repeating

pi, π

square root
radicand
perfect square

number line

equal
=
less than
<
greater than
>
less than or equal to
≤
greater than or equal to
≥
compound inequality
(ex. a < x < b)

ascending order
descending order

estimation
place value
benchmark values

C.   The hypotenuse of a right triangle is $\sqrt{32}$. Simplify this radical expression.

D.   The hypotenuse of a right triangle measures 18 and one of the sides has a measure of 15. Give the exact measure of the missing side.

  a) 99   b) 33

  c) $3\sqrt{61}$ d) $3\sqrt{11}$

E.   Which of the following expressions are irrational?

  a) $\sqrt{25}$ b) 0.010010001...   c) $^4/_7$

  d) $\sqrt{8}$ e) 0.$\overline{3}$ f) π

F.   Graph the following real numbers on a number line.

  a) $\sqrt{32}$ b) 8.2   c) $\sqrt{210}$ d) - 4

  e) - $^2/_3$ f) $\sqrt{6}$ g) 2 $^7/_8$

G.   If a, b and c are real numbers, a and c are non-negative and $\sqrt{a}$ = $b\sqrt{c}$, what must be true about a, b, and c?

H.   If a, b and c are real numbers, b is non-negative, and $a\sqrt{b}$ = c, what must be true about the values for a and b if:

  • $c = 0$
  • $c < 0$
  • $c > 0$

I.   Given: a, b and c are real numbers, b is non-negative, and $a\sqrt{b}$ = c. Without using a calculator, what must be true about the value of the missing variable?

a	b	c
$a = 5$		$c > 50$
$a < 3$		$c = 12$
	$b < 49$	$c = 56$
$a = 4$	$b < 81$	

J.   Locate the following irrational numbers between two consecutive integers.

  a) - $\sqrt{14}$ b) $\sqrt{21}$ c) - $\sqrt{75}$

  d) - $\sqrt{118}$ e) $\sqrt{232}$ f) $\sqrt{600}$

Grade Eight: Number and Operations

1.02 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

A. In basketball, the equation $P = 2x - x^2$ describes a player's theoretical chance of scoring at least one point in a two-shot free throw situation. P is the player's chance of scoring and x is the player's current free throw shooting percentage. If Ray is a 39% free throw shooter, what is his chance of scoring at least one point the next time he is in a two-shot situation?

guess and test
make a table/chart/
graph
make a diagram/picture
make an organized list
work backwards
work a simpler problem
find a pattern
extraneous information

B. If 10% of a number is 6, what is 35% of the same number?

C. Jane wants to fence her 25 ft. by 35 ft. garden plot. She plans to put a post every 5 feet along the perimeter. How many fence posts will she need?

D. You spend 30 minutes watching TV and 3/4 of an hour studying for each of your two tests. You then decide to take a 15-minute snack break. If you finish studying at 5:30 p.m., what time did you start?

2.01　Determine the effect on perimeter, area or volume when one or more dimensions of two- and three-dimensional figures are changed.

length
width
base
height

radius
diameter
circumference
pi, π

polygon
prism
cube
cylinder

A.　　Joy has a 2 gallon tank that houses two goldfish. The base of the tank is 14 inches by 6 inches. The height of the tank is 8 inches. For her birthday she was given a new fish tank that had a base 28 inches by 12 inches and a height of 16 inches. Approximately how many gallons of water would the new tank use?
Joy's brother has a tank that holds x gallons of water. If each dimension were doubled, how many gallons, in terms of x, would his larger tank hold ?

B.　　John and Mary each have a garden in the shape of a square. If the area of John's garden is four times the area of Mary's garden, how do the perimeters of the two gardens compare?

C.　　A company manufactures two different cylindrical containers for storage of pool chemicals. If the radius and height of the larger container are twice the radius and height of the smaller container, by what factor does the volume increase?

D.　　A square scarf is folded in half to form a rectangle. If the resulting rectangle has a perimeter of 15 inches, what are the area and perimeter of the square scarf?

E.　　Jenni made a small table for a class project. The top of the table is triangular in shape. Her mother wants her to make a larger table. If she changes the dimensions so that both the base and height of the triangle are tripled, how will the area of the table top change?

F.　　A cube has sides that measure 8 inches. If the length of one of the sides is increased by 25 percent, what is the percentage increase in the volume?
(From SREB publication *Getting Students Ready for Algebra I: What Middle Grades Students Need to Know and Be Able to Do*)

# 2.02 Apply and use concepts of indirect measurement.

> *To achieve this objective, students should be able to use ratios, similar figures, and the Pythagorean Theorem to determine measurements that are difficult or inconvenient to find with direct measurement.*

A. A billboard is 18 feet high and casts a 24 foot shadow. At the same time, a flagpole next to the billboard casts a 60 foot shadow. Find the height of the flagpole.

B. Jesse, who is 6 feet 2 inches tall, wants to determine the height of the oak tree located in the school yard. At 10:00 a.m. his shadow measured 2 feet 6 inches. What is the height of the tree if its shadow at 10:00 a.m. is approximately 35 feet?

C. A 13-foot rope is tied 5 feet from the top of a pine tree. It is anchored 7 feet from the base of the tree. How tall is the pine tree?

similar figures
corresponding parts
ratio
proportion
scale drawings

Pythagorean theorem

13 foot rope

7 feet
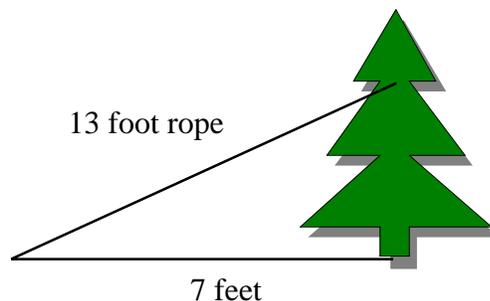
D.   An engineer is drawing plans for a walkway across a pond. She is unable to measure the distance across the pond directly so she plans on using properties of similar triangles to determine the distance. Using wooden stakes and rope, she sets up two similar triangles as shown below. If Δ ABC is similar to triangle Δ DEC, determine the distance across the pond.

16 m

A   B

12 m   10 m

C

32.5 m   39 m

?

E   D
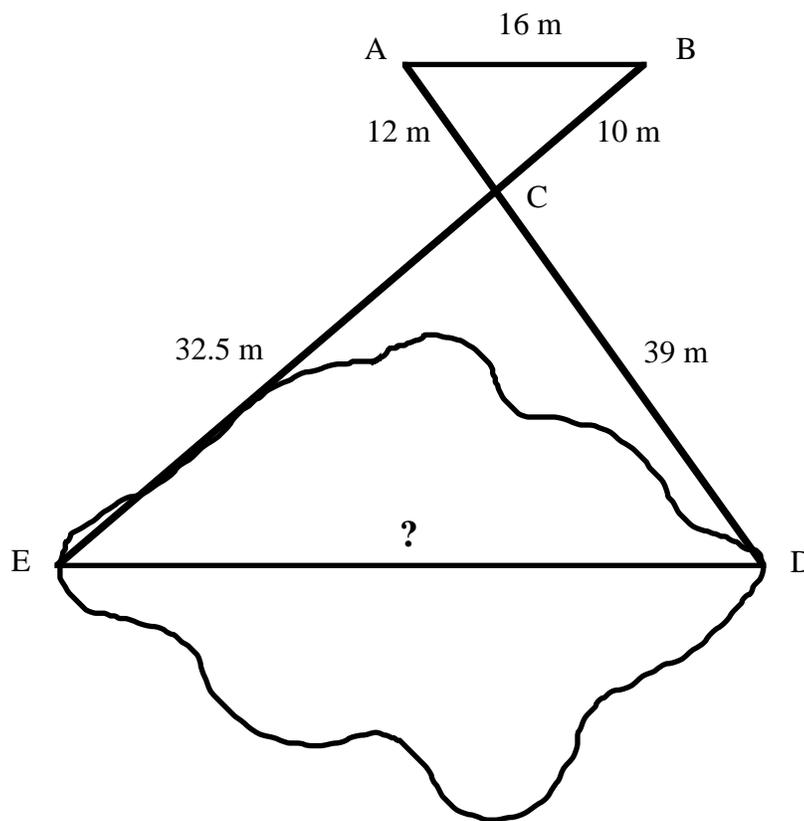
E.   Toni's dad works in construction and knows that safety is of utmost concern when placing a ladder against a house. If he is using a 24-foot ladder, he makes sure that he has the foot of the ladder on flat ground 8 feet from the house. In order to keep this same ratio, how far from the house should he place an 8-foot ladder? If he is using the 24-foot ladder as described above, what is the vertical distance from the top of the ladder to the ground?

3.01 Represent problem situations with geometric models.

A.　　Carlos bought a pizza with an area of approximately 201 in². He paid $8.99 for the pizza. Tameka bought two pizzas, each with an area of approximately 133 in². She paid a total of $10.99 for the two pizzas. Pizzas are usually sold by the measurement of the diameters. What is the diameter of each pizza (to the nearest half-inch)? Which pizza is the better buy based on the number of square inches per pizza? Show the model you used to find the solution.

B.　　At a carnival, the dart game uses a square game board with a side length of 8 feet. In order to win the game, the dart you throw must land in one of four non-overlapping circles located on the board. Each circle has a diameter of 18 inches. If a dart lands on the board, what is the probability of it landing in one of the four circles? Show the model you used to find the probability.

C.　　A 10 foot by 14 foot wall contains two window openings that each measure 3 feet by 5 feet. Make a scale drawing of the wall and windows and find the percent of the wall that will need to be painted.

D.　　Six students in a study group decide to exchange telephone numbers so they can contact each other between study sessions. How many exchanges can occur? Use a model to find the number of exchanges.

geometric probability
area
volume
cylinder
cube

3.02   Apply geometric properties and relationships, including the Pythagorean theorem, to solve problems.

To achieve this objective, students should be able to:

- *Apply ratios, similarity, and proportional reasoning to solve problems.*

- *Apply formulas for finding area, perimeter/circumference, and volume to solve problems.*

- *Apply properties of quadrilaterals to solve problems.*

right triangle
hypotenuse
leg
Pythagorean triple

complementary angles
supplementary angles

perimeter
area

square
square root

irrational number

A.   Mr. Adams has enclosed a square flower bed in his yard that measures 5 feet on a side. He decides to partition the square into two triangular parts by placing a row of bricks, each 1 foot long, diagonally through the square. How many bricks will he need?

B.   A television tower has a guy wire attached 6 meters from the top of the tower and secured to the ground 40 feet from the base of the tower. If the guy wire is 50 meters in length, how tall is the tower?

C.   Josh marked off a section of the playground in the shape of a right triangle. If the hypotenuse of the triangular section measures 10 meters and one of the legs measures 6 meters, what is the area of this section of the playground?

D. Use the diagram below to determine the distance across Lake James.

16 miles   20 miles

A   B

Lake James
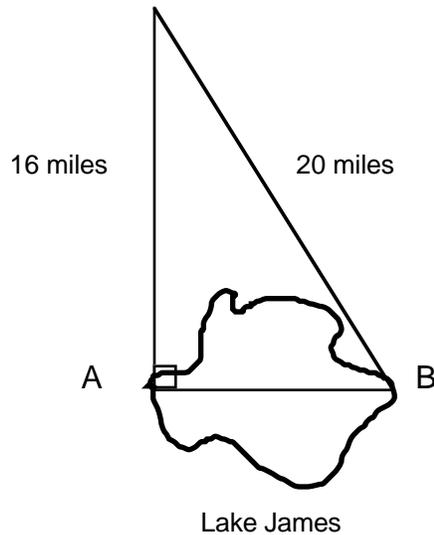
E. The perimeter of rectangle *BCDE* is 32 inches. If the length of the rectangle is 3 times its width and the area of triangle *ABE* is 6 square inches, what is the perimeter of trapezoid *ABCD*?
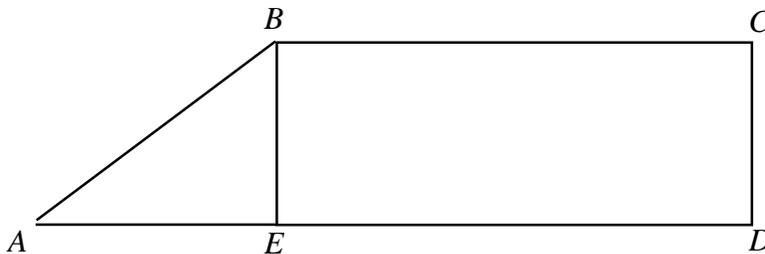
B   *C*

A   *E*   *D*

(From SREB publication *Getting Students Ready for Algebra I: What Middle Grades Students Need to Know and Be Able to Do*)

F. Two students are using a measuring tape to find the length of a room. The tape reads 13 feet 6 $\frac{1}{4}$ inches; however, the student at one end is holding the tape 9 inches higher than the student at the other end. What is the difference between their measurement and the true length?

G. Figure *ABCDE* is a convex polygon with such that:
$m\angle A = 6x°$,
$m\angle B = (4x + 13)°$,
$m\angle C = (x + 9)°$,
$m\angle D = (2x - 8)°$, and
$m\angle E = (4x - 1)°$.
What are the degree measures of each of the angles?

H. Triangle *ABC* is similar to triangle *DEF*. If the lengths of the sides of the two triangles are in the ratio of 2:1, what is the ratio of the area of the larger triangle to the area of the smaller triangle?

I. A cereal box has the following dimensions: 27 cm x 18 cm x 6 cm. To become more environmentally friendly, a company is trying to decide if it should change the shape of the cereal box to a cylinder with a height of 27 cm and a diameter of 18 cm.   Would it hold the same amount of cereal? Why or why not?
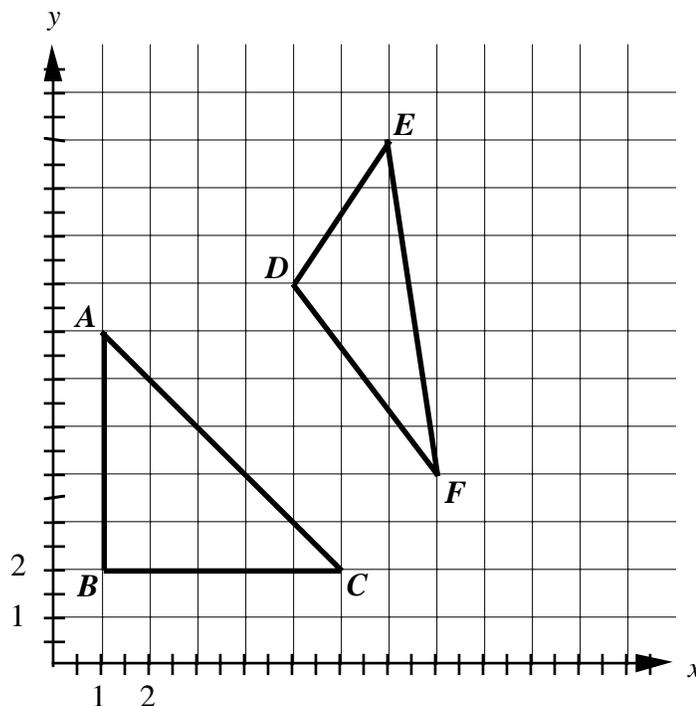
J. Give the exact length of side *AC* for triangle *ABC*.
a) 50   b) 25

c) $5\sqrt{2}$ d) $\sqrt{10}$

K. Determine the perimeter of triangle *DEF*. Give your answer to the nearest tenth of a unit.

Grade Eight: Geometry

L. Complete the following chart of Pythagorean triples (three positive integers that satisfy the condition that $a^2 + b^2 = c^2$).

a	*b*	*c*
3		5
	12	13
7		25
8	15	
21		29

M. Using the fact that 3, 4, 5 is a Pythagorean triple, which of the following are also Pythagorean triples?

(5, 6, 7)
(6, 8, 10)
(12, 16, 20)
(30, 40, 50)
(35, 45, 55)

3.03 Identify, predict, and describe dilations in the coordinate plane.

> *To achieve this objective, students should be able to:*
>
> - *Recognize that a transformation of the form $(x', y') = (ax, ay)$ is a dilation that enlarges or reduces the figure by a factor of a.*
>
> - *Use sample notation (see side bar) to describe dilations.*

transformation
enlargement
reduction
shrink
stretch
scaling
scale factor
image
pre-image

notation:
$\Delta ABC \longrightarrow \Delta DEF$
$\Delta ABC \longrightarrow \Delta A'B'C'$
$A \longrightarrow A'$
$(x, y) \longrightarrow (x', y')$
$(x', y') = (ax, ay)$

similarity
ratio
proportion
corresponding parts

coordinate plane

A. On graph paper, draw one of your initials using block letters. Now create a dilation of this letter using a scale factor of two. How does the area of the dilation compare to the area of the original letter?

B. How do the images of dilations with scale factors greater than one compare to their pre-images? How do the images of dilations with scale factors less than one but greater than zero compare to their pre-images?

C. Triangle *ABC* with coordinates *A* (2, 4); *B* (2, 2); and *C* (4, 2) is transformed with the new coordinates of triangle *A'B'C'* as follows: *A'* (4, 10); *B'* (4, 4); and *C'* (10, 4). Is this transformation a dilation? Explain.

D. John claims that figure *A'B'C'D'* is a dilation with a scale factor of one-half. He made an error when listing the coordinates of the points. Identify the error and make the appropriate correction. Explain your reasoning.

I.	*A* (4, 2)	*A'* (2, 1)
II.	*D* (-6, -4)	*D'* (-3, -2)
III.	*C* (3, -8)	*C'* (1, -4)
IV.	*B* (-6, 1)	*B'* (-3, 0.5)

E.	Plot the given points on a coordinate plane. Connect the points in order and connect the last point to the first one.
	(-6, 8), (2, 8), (8, -4), (-6, -4)

	a. What polygon is formed?
	b. Graph the image of the polygon after a dilation with scale factor of 0.25.
	c. How does the area of the image compare to the area of the original polygon?

F.	The image of figure *QRST* is figure *Q′R′S′T′*. What is the scale factor for this dilation?
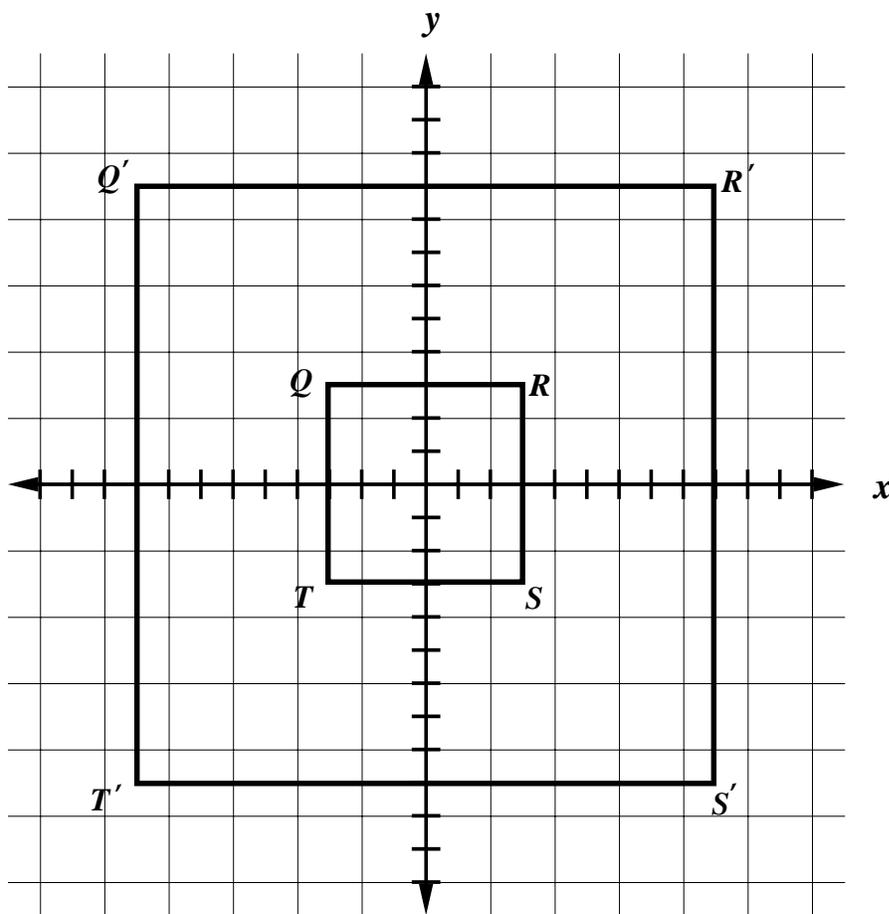
F.	Triangle *KLM* with coordinates *K* (-3, -2); *L* (0, 4); and *M* (4, 2) is transformed according to the rule: $(x′, y′) = (3x, 3y)$. Graph the image of triangle *KLM* and give the coordinates of its vertices.

4.01   Collect, organize, analyze, and display data (including scatterplots) to solve problems.

To achieve this objective, students should be able to:

- *Choose appropriate scales and accurately plot and label paired-data.*

- *Analyze the relationship among pairs of data in a scatter plot.*

- *Determine if a given scatter plot has a positive relationship (correlation), a negative relationship, no relationship, or a constant relationship.*

- *Use scatter plots to make predictions and draw conclusions.*

- *Recognize trends in data from a scatter plot.*

coordinate plane
dependent variable
independent variable

trend line
line of best fit
trend/correlation
positive correlation
positive relationship
negative correlation
negative relationship
no correlation
no relationship
constant relationship

A.   When creating a scatterplot, on which axis should the independent variable be graphed? The dependent variable? Using the table below, create a scatterplot:

MidView Basketball Team Data

| Player	Shots Attempted	Shots Made
A	240	160
B	200	110
C	130	50
D	80	60
E	76	70
F	36	20
G	20	10

Which is the independent variable?   Which is the dependent variable?

Is there a positive, negative or no correlation for the data?   Draw a line of best fit for the data.

B.   Determine whether a scatter plot for the following situations would have a positive correlation, negative correlation, or no correlation.

1. Amount of time spent studying and students' test averages

2. Outside temperature and heating costs

3. Your savings account number and the balance in the account

C.   Each point on the graph below represents a different bag of sugar.

1. Which bag is heaviest?

2. Which bag is the least expensive?

3. Which bags are the same weight?

4. Which bags cost the same price?

5. Which has the better value for the money, *F* or *C*? How do you know?

6. Which has the better value for the money, *B* or *C*? How do you know?

7. Which bags give the same value for the money? How do you know?
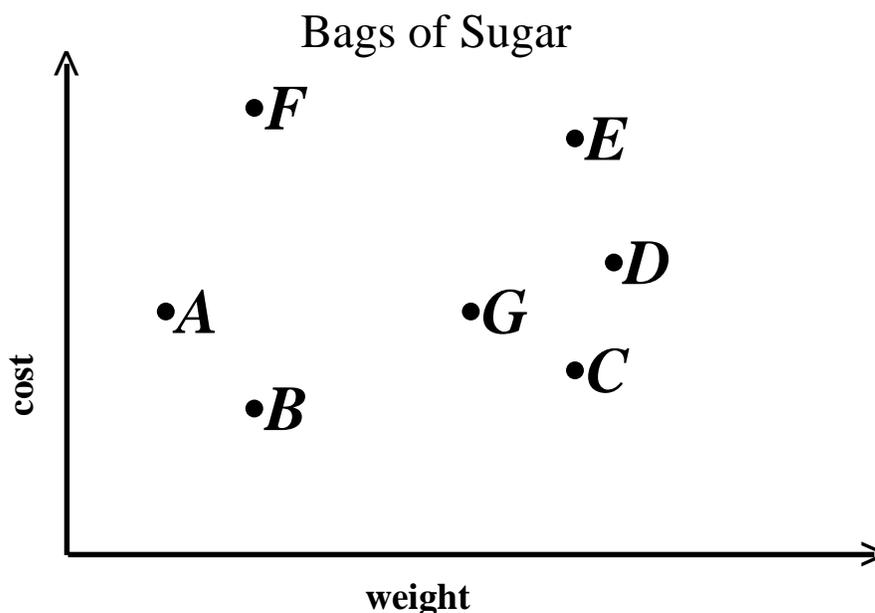
Bags of Sugar

Grade Eight: Data Analysis and Probability

4.02 Approximate a line of best fit for a given scatterplot; explain the meaning of the line as it relates to the problem and make predictions.

> *To achieve this objective, students should to be able to use <u>informal</u> methods to find lines of best fit.*

coordinate plane
dependent variable
independent variable
y-intercept
x-intercept
ordered pair

slope (rate of change)
correlation/trend
positive correlation
positive relationship
negative correlation
negative relationship
no correlation
no relationship
constant relationship

A.   Measure the circumference and diameter of 15 to 20 round objects, recording the results in a table. Then make a scatterplot (with diameter on the horizontal axis) and determine the line of best fit. Identify the slope of this linear relation. Based on your results, what would be the circumference of an object with diameter of 13 inches?

B.   A salesman at the local car dealership estimated the number of SUVs that he would sell for a ten month period. He also recorded the actual number of sales he made. The results are given below. Use a scatterplot to display this data. Draw a line of best fit for the scatterplot. Without using a calculator, approximate the equation of this line. Is there a positive, negative, or no correlation for these data?

| Month	Estimate of SUVs sold	Actual number of SUVs sold
1	2	3
2	5	4
3	5	3
4	9	11
5	12	9
6	8	10
7	13	16
8	14	13
9	3	5
10	16	15

4.03　Identify misuses of statistical and numerical data.

To achieve this objective, students should be able to:

- *Interpret the effects of changing the scale on a graph.*

- *Identify biased questions.*

- *Distinguish between representative, biased and insufficient sampling.*

A.　Explain why the following graph is misleading. Correct the graph so that a more accurate impression of the data is conveyed.

Four Busiest U.S. Airports
2002 Passengers
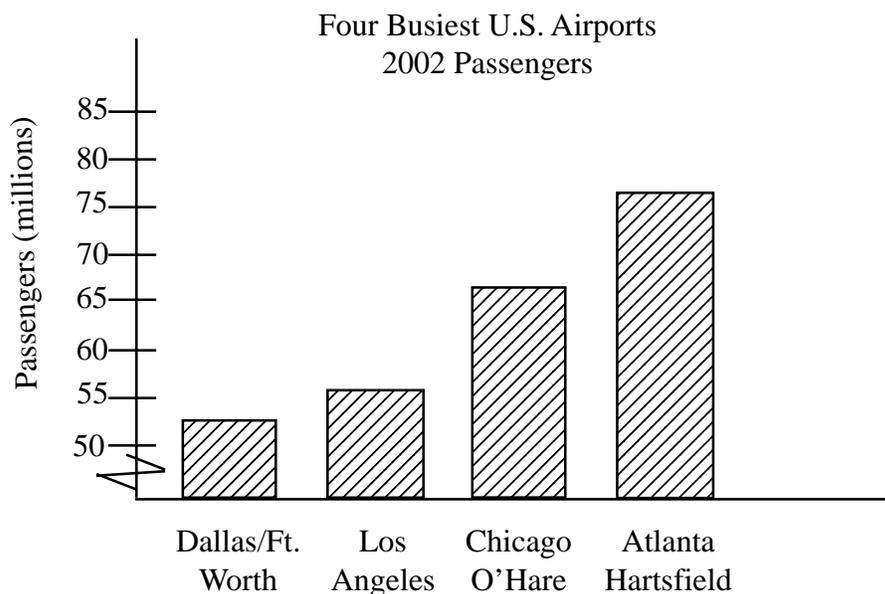
Source: www.infoplease.com

measures of central
tendency
mean
median
mode

range
outlier

sample
representative sample
biased sample
sample size

scale

B.   The middle school cafeteria manager is considering adding cheeseburgers to the daily menu. He wants to get feedback from a sample population of students and decides to survey the first 10 students in each 8ᵗʰ grade homeroom. Would the data collected be representative of the entire school population? Why or why not?

C.   A market researcher randomly selects 12 people to taste-test three leading brands of chips labeled *A*, *B*, and *C*. Of these, six chose brand *A*, three chose brand *B*, and three chose brand *C*. An ad for brand *A* states: "Preferred 2 to 1 over the leading brands".
Is this a valid statement? Explain.

D.   In a poll taken at a middle school, 38% of the students indicated a preference for rap music, 32% preferred alternative music, and 30% like country music best. A report stated that most of the students at the school liked rap best. Is this an accurate statement? Explain.

E.   A small business placed a help wanted ad that stated that its average salary is $35,800. The actual salaries of the five employees are: $100,000, $16,000, $22,000, $20,000 and $21,000.
Is this statement about the average salary correct? How might this statement mislead a potential employee?

5.01   Develop an understanding of function.

a) Translate among verbal, tabular, graphic, and algebraic representations of functions.

A.   At the beginning of the summer, Beth has $2, John has $30, and Sam has $14. Beth and John each earn $3 per day for baby-sitting Sam. John spends $5 each day at the movies while Beth spends $2 each day playing video games. Sam stays at home and saves his money. Make a table showing how much money each person will have at the end of each day for two weeks. Make a graph of the data. On which day would John have $6? Which person has the most money at the end of the two weeks? On which day would John and Sam have the same amount of money? Which person's graph shows a positive relationship? a negative relationship? a constant relationship? How much money would you expect Beth to have on the twentieth day? If Beth decided to spend only $1 on video games instead of $2, how would her graph look different?

B.   The data in the table represents the cost of dance tickets.

Number(*n*)	1	3	4	6
Cost(*c*)	6	18	24	36

Write an equation representing this function. Make a graph of the equation. Determine the slope. Explain the meaning of the slope for this function.

C.   An electrician charges customers a basic rate of $22 for making a service call plus $35 for each hour worked. Write an equation to represent the total charge, *C*, if *h* represents the number of hours worked. Explain the meaning of the *y*-intercept in terms of this situation.

D.　　On a recent backpacking trip up a mountain trail, Jim noticed that it seemed to get colder the further up the mountain he hiked from about 9 a.m. until about 3 p.m. He thought that it should get warmer in the afternoon hours, but the temperature dropped as indicated in the data below. Decide which is the dependent and independent variable, graph the data, draw the line that represents the relation, and determine the slope of the line. Explain the meaning of the slope for this relation. Based on this data, what would the temperature be at an elevation of 6,000 feet? What trend or relationship appears to exist between increasing elevation and temperature?

Elevation in feet	Temperature (Fahrenheit)
1,000	70°
2,000	69°
2,500	68.5°
4,000	67°
5,000	66°
5,500	65.5°

properties of equality
commutative property
associative property
distributive property
identities

Students need to be familiar with a variety of notations for multiplication:
　　a x *b*
　　a • *b*
　　a(*b*)
　　ab

graph
slope (rate of change)
intercepts
y-intercept
x-intercept

E.　　The following is a profit/loss graph for a sidewalk lemonade stand. Explain the meaning of the *x*- and *y*-intercepts and the slope as it relates to this situation.

Lemonade Sales

Grade Eight: Algebra
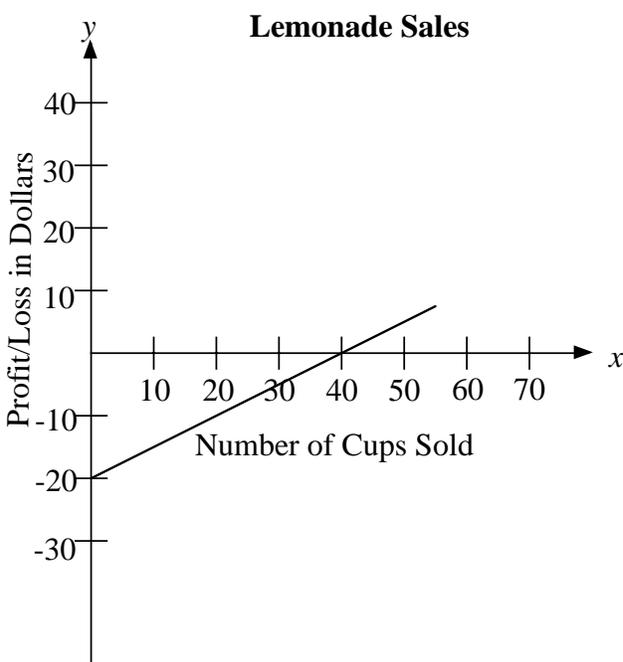
F.   f(x) = -2x + 4
Evaluate this function for { x: x = 0, 1, 2, 3, 4, 5}
Identify the values of the domain.
Identify the values of the range.

G.   Jose rides his bicycle to school. He leaves his house at 8:00 a.m.
and travels at a steady rate for ten minutes. He then goes up a hill for
five minutes. At the top of the hill he stops for water. At 8:20 a.m. he
continues on his way traveling downhill for ten minutes. He gets to
school and parks his bicycle. Create a graph to represent Jose's bicycle
trip to school and explain your graph.

b) *Identify relations and functions as linear or nonlinear.*

A.   Identify each of the following as linear or nonlinear.
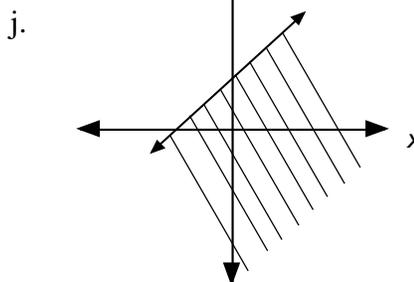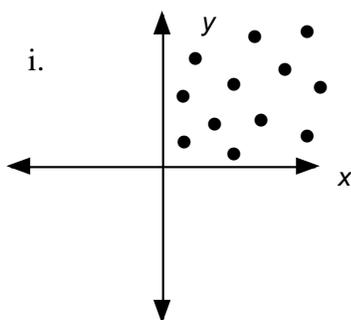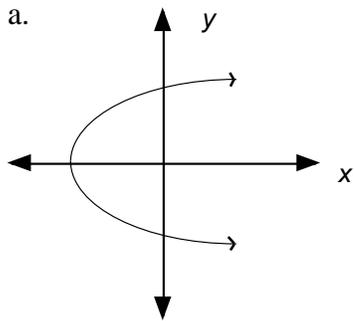a. $y = 3x + 4$ b. $y = x^2 + 1$ c. $y = -5$ d. $x + y \leq 17$

e.

f.
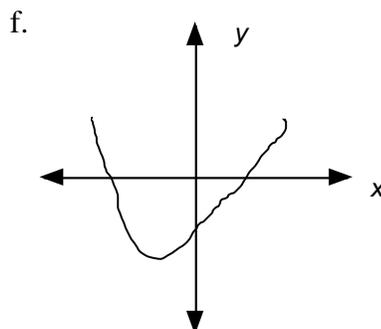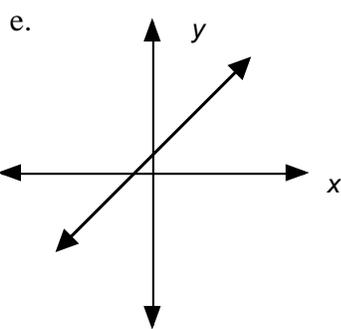
g. $y = \dfrac{1}{x + 2}$ h.
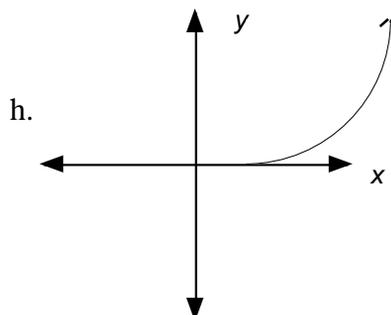
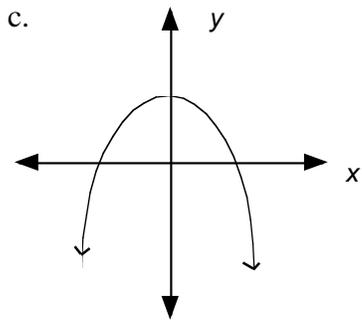slope (rate of change)
y-intercept
x-intercept
vertical line test

i.

j.

Grade Eight: Algebra

B. Determine which of these relations are functions. Explain your reasoning.

a.

b.
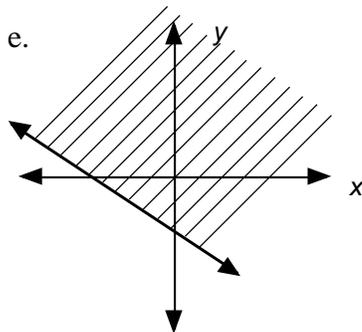
x	y
16	4
16	-4
36	6
36	-6

c.

d. $\{(0, 2), (1, 3), (2, 5), (3, 6)\}$

e.

c) Find, identify, and interpret the slope (rate of change) and intercepts of a linear relation.

To achieve this objective, students should be able to:

- *Understand slope as the constant rate of change in a line.*

- *Find the slope of a line (rise/run) given a set of ordered pairs, a table of values, the graph of a line, or the equation of a line.*

- *Distinguish between lines with no slope (vertical) and lines with zero slope (horizontal).*

- *Identify where a line crosses the x-axis (x-intercept) and the y-axis (y-intercept).*

coordinate plane
y-intercept
x-intercept

function

A.   The total amount paid on a charge card can be represented by the equation $y = 0.18x + 25$, where there is an 18% annual interest rate plus a $25 annual fee.
        a) Graph the linear equation on a coordinate plane.
        b) Determine the slope and explain its meaning.
        c) Determine the *y*-intercept and explain its meaning.

B.   Jack and Jill started a lemonade stand by borrowing $10 worth of supplies from their mom. They charged $0.50 per cup of lemonade. Write an equation to represent their profit.
        a) Graph the linear equation on a coordinate plane.
        b) Determine the slope and explain its meaning.
        c) Determine the *x*- and *y*-intercepts and explain their meaning.

C.   MovieTime Video charges a $10 yearly membership fee and $5 to rent each video. Using the information given in the table regarding yearly cost, determine the equation to represent this function. Draw a line of best fit for the data. Determine the slope and explain its meaning. Determine the *y*-intercept and explain its meaning. Determine the cost for renting 60 videos.

# of Videos	0	10	20	30	40
Cost	$10	$60	$110	$160	$210

D. Students in a local university will see their student fees increase from $954 in 2002 to $982 in 2003. Assuming this is a linear trend, how much will students be paying for student fees in 2006? What is the slope of the linear relation which describes this growth in fees?

E. Describe the change in the graph of $y = ax + 7$ as a increases in value.

F. Describe the change in the graph of $y = 2x + b$ as b decreases in value.

d) *Interpret and compare properties of linear functions from tables, graphs, or equations.*

To achieve this objective, students should be able to:

- *Use slope to identify parallel lines (same slope).*

- *Determine if the graph of two linear functions has no points in common (parallel lines), has one point in common (intersecting lines), or has all points in common (same line).*

- *Use knowledge about linear relationships to solve problems.*

linear function
non-linear function
domain
range
increasing
decreasing

intercepts
slope

parallel lines
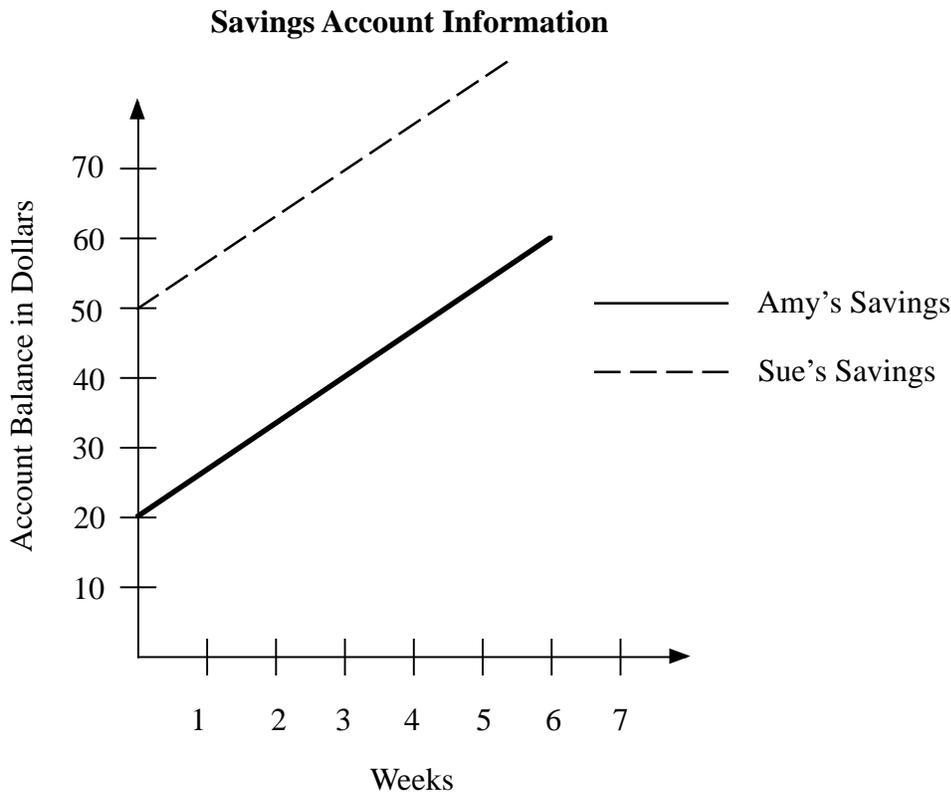intersecting lines
perpendicular lines
collinear points

A.   The table below gives the number of diagonals, $f(n)$, that can be drawn from one vertex in a polygon of n sides.

n	4	5	6	7
$f(n)$	1	2	3	4

a. What conclusion can be drawn from the information in the table? Draw sketches to justify your conclusion.

b. Using function notation, write an equation that represents the number of diagonals that can be drawn from one vertex of a polygon of n sides.

c. What can you conclude about the total number of different diagonals that can be drawn in a polygon of n sides?

B.   United Gameware is a company that makes games for PCs. For the last five years the equation $y = 3.25x + 10.75$ modeled the growth in value of the company's stock. With an initial offering price of $10.75, $3.25 is the average annual change in value of the stock, x is the number of years since the initial offering, and y is the value of the stock. A competitor, FedGames, issued its stock with the same initial value but only grew $1.95 a year in value. What would FedGames' linear model look like? After five years, which company's stock is worth more? by how much? Suppose United Gameware began losing $0.85 per year in value after the fifth year. What would the linear equation look like then?

C.   The following graph illustrates the money in Amy's and Sue's savings accounts over a 6-week period. Using the graph, decide if the two girls will ever have the same amount of money in their savings accounts. Explain your reasoning.   If not, how could you change one of the girl's savings plan so that the two girls will eventually have the same amount of money?

Savings Account Information

D.   For each of the following pairs of functions, determine if the graph contains parallel, intersecting or the same lines.

a. $x + y = 8$
$5y + 5x = 12$

b. $4x - 3y = 10$
$8y + 6x = 12$

c. $6x - 9y = 36$
$y = \dfrac{2}{3} x - 4$

d. $3x - 2y = 12$
$y = \dfrac{3}{2} x - 6$

e. $6x - 5y = 18$
$2y + 7x = 8$

5.02   Write an equation of a linear relationship given: two points, the slope and one point on the line, or the slope and y-intercept.

equation
slope-intercept form
$f(x) = ax + b$
point-slope form
$y - y_1 = a(x - x_1)$
standard form
$Ax + By = C$

A.   Determine the equation for each of the following:
a. The line containing point *A* (-4, 5) and point *B* (3, -6)
b. The line through the point *C* (5, -2) with slope of $^3/_5$
c. The line with the *y*-intercept at (0, 18) and slope of $^{-5}/_2$
d. The line with *x*-intercept of 4 and *y*-intercept of 2.

B.   Graph and give the equation of each of the following lines:
a. Line *CD* which includes points *C* (-6, 4) and *D* (2, -7)
b. Line *m* with *y*-intercept at (0, -5) and slope of $^2/_3$
c. Line *AB* through point *A* ($^3/_4$, 6) with slope of 2
d. Line *p* through point *B* (4, -2) with slope of 0

C.   The number of licensed pilots in the United States grew from 622,000 in 1996 to 640,000 in 1999. Assuming a constant annual increase in the number of pilots, how many pilots could we expect to have by 2007? Write a linear equation to represent this relationship.

D.   A man is 180 miles from home and is driving home at a speed of 45 mph. Write an equation to determine his distance from home at any point during his trip.

E. The 8th grade class is planning a barbeque picnic. They have a total of $85 to spend on chicken and hamburger. Chicken sells for $3 a pound and hamburger sells for $2.50 a pound. Write an equation in standard form to indicate the amount of chicken and hamburger they can buy.

5.03 Solve problems using linear equations and inequalities; justify symbolically and graphically.

To achieve this objective, students should be able to:

- *Interpret situations mathematically and write appropriate algebraic expressions to represent them.*

- *Represent and interpret single-variable equations/inequalities on a number line and two-variable equations/inequalities using the coordinate plane.*

A.. When I was 8 years old my father was 32 years old, which was four times my age. Now he is twice my age. How old am I?

use of formulas

coordinate plane

B. The Tentor family of five is going camping at Watagua State Park. The State Park charges a one-time flat fee of $3.00 per person. In addition, there is a nightly rate. The Tentors were charged a total of $63.00 for four nights. Write and solve an equation to determine the nightly rate. The Tentor's neighbors have decided to join them. They have three people in their family and can stay an extra night. What is their neighbors' cost?

x-intercept
y-intercept

less than
$<$
greater than
$>$
less than or equal to
$\leq$

C. The graph of the line $3x + 4y = 48$ intersects both the *x*-axis and *y*-axis forming a triangle. Find the area of this triangle.

greater than or equal to
$\geq$
compound inequality
$a < x < b$

D. A new band, The Topsail Pirates, has just made a CD. They recorded it themselves at a cost of $8,000. They have been offered a contract by a recording company to produce and promote it. The company is offering them $5,000 and 16 percent of total sales. What will the minimum amount of sales need to be in order for the band to recover the expense of recording the CD?

E.   The power company uses two different rates to calculate a monthly power bill. For July-October, the basic customer charge is $6.75 plus $0.08065 per kilowatt-hour. For November-June, the basic customer charge is $6.75 plus $0.07065 per kilowatt-hour. A sales tax of 3% is added for the final charge. If the May and September bills are both $127, what is the difference in the amount of power (kilowatt-hours) used each month?

F.   A video store rents new releases for $5 each and family special videos for $2 each. Steve has $15 to rent videos.
   a. Give an inequality that can be used to determine the number
       of each type of video Steve can rent.
   b. Graph the inequality.
   c. Using the graph, give three possible combinations of videos
       Steve could rent.

G.   The number, n, of CDs in Harper's CD holder is described by the following inequality: $12 < n - 5 < 26$. Which of the following sets contains values that could represent the number of CDs in the holder?
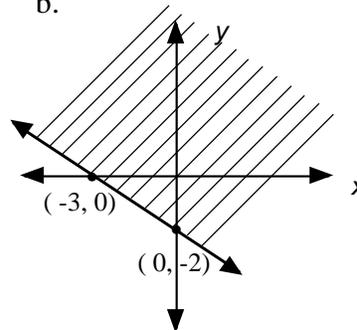
   a. {12, 13, 14, 15,...,26}
   b. {13, 14, 15, 16,...,25}
   c. {17, 18, 19, 20,...,31}
   d. {18, 19, 20, 21,...,30}

H.   Write an inequality for each of the following:
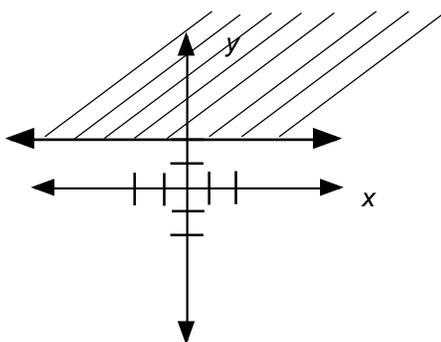
a.

b.

5.04　Solve equations using the inverse relationships of addition and subtraction, multiplication and division, squares and square roots, and cubes and cube roots.

A.　A group of students is decorating for the school dance. They plan on attaching thirty-five crepe paper streamers to the top of a 4-foot pole and securing them to the floor at a distance of 3 feet from the base of the pole to form a circle. What should be the length of each of the streamers that they cut for this project? If there are 9 yards of crepe paper per roll, how many rolls of crepe paper will they need to purchase for this project?

B.　For each of the following, solve for the variable indicated.
　　a. $P = 2l + 2w$; solve for w
　　b. $V = lwh$; solve for h
　　c. $A = \pi r^2$; solve for r
　　d. $V = {}^1\!/_3\, Bh$; solve for B

C.　 Jenni has a large box in the shape of a cube which she uses to store her summer clothes. If the volume of the box is 27 cubic feet, what is the length of an edge of the box?

D.　Mario ordered three pizzas to share with a group of friends.   To the nearest square inch, the area of the pepperoni pizza is 79 square inches, the area of the veggie pizza is 154 square inches and the area of the pineapple pizza is 201 square inches. Determine, to the nearest inch, the diameter of each of the pizzas.

E.　The formula for the surface area of a sphere is $A = 4\pi r^2$. Find the radius if the surface area is 20 in².

F.　For each of the following, solve for x:

　　a. $2.3x - 4.7 = -10.7x + 27.8$
　　b. $3(x + 5) = 2(x + 3) - 11$
　　c. $-4(x - 8) = 18 + 3x$
　　d. $3 - \dfrac{1}{2}(x + 7) = 18$